Physics 10154 - Exam #4a

Partial credit will be given provided you show all work and are solving parts of the problem correctly. Points will be deducted if you don't show your work (or if some parts are incorrect) even if you get the right answer. Clearly indicate your answer with a circle or box and remember to include correct units and significant figures.

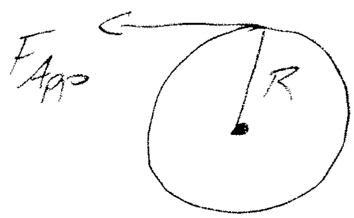
1. A 250 kg merry-go-round in the shape of a uniform, solid cylinder of radius 1.4 meters is set in motion by wrapping a thin rope around the rim of the disk and pulling on the rope so that the rope is tangent to the rim of the disk. A frictional torque of 12 N-m opposes any motion of the merry-go-round, which is horizontal.

If the merry-go-round starts from rest and reaches an angular speed of 1.0 rev/sec in 5 seconds, answer the following:

a) (25 pts) What is the applied force pulling on the rope?

$\Delta\theta = ?$
$\omega_0 = 0$
$\omega = 6.28$ rad/sec
$\alpha = ?$
$t = 5.0$ s

$\alpha = 1.256$ rad/s^2



$I = \frac{1}{2}MR^2$
$= 245$ kg m^2

$\Sigma\tau = \tau_{App} - \tau_{Fric} = I\alpha$

$= +RF_{App}\sin 90 - 12 = 245\alpha$

$\Rightarrow (1.4)F_{App} = 245(1.256) + 12$

$$\boxed{F_{App} = 230\ N}$$

b) (15 pts) The instant the merry-go-round reaches its final speed, the rope is released (and we can now ignore friction), and a 55 kg child jumps onto the merry-go-round with no initial angular speed. What is the new angular speed of the merry-go-round?

$$I_{1i} = I_{1f} = 245 \text{ kg}\cdot\text{m}^2$$

$$I_{2i} = I_{2f} = MR^2 = (55)(1.4)^2 = 107.8$$

$$\omega_{1i} = 6.28 \text{ rad/s} \qquad \omega_{1f} = \omega_{2f} = \omega_f$$

$$\omega_{2i} = 0$$

$$I_{1i}\omega_{1i} + I_{2i}\omega_{2i} = I_{1f}\omega_{1f} + I_{2f}\omega_{2f}$$

$$245(6.28) + (107.8)(0) = (245 + 107.8)\omega_f$$

$$\boxed{\omega_f = 4.4 \text{ rad/s}}$$

c) (10 pts) How much kinetic energy is lost when the child jumps onto the merry-go-round?

$$K_f = \frac{1}{2} I_1 \omega_{1i}^2$$

$$= \frac{1}{2}(245)(6.28)^2 = 4831 \text{ J}$$

$$K_f = \frac{1}{2} I_1 \omega_f^2 + \frac{1}{2} I_2 \omega_f^2$$

$$= \frac{1}{2}(245 + 107.8)(4.4)^2 = 3415 \text{ J}$$

$$\boxed{\Delta K = -1400 \text{ J}}$$

2. A 25-kg block of wood floats on the surface of a water tank (water has a density of 1000 kg/m3) with 32% of the block's volume exposed.

a) (15 pts) What is the density of the block of wood?

Block is 68% submerged $\frac{V_f}{V_o} = 0.68$

$$\Sigma F_y = F_B - F_{grav} = 0$$

$$\rho_f V_f g - \rho_o V_o g = 0$$

$$\rho_f V_f = \rho_o V_o$$

$$\rho_o = \rho_f \frac{V_f}{V_o} = (1000)(0.68) = \boxed{680 \text{ kg/m}^3}$$

b) (15 pts) How much additional mass can be placed on the block of wood before it begins to sink?

To sink, need $\Sigma F_y = F_B - F_{grav,block} - m_{extra} g = 0$

$$V_{obj} = \frac{25 \text{ kg}}{680 \text{ kg/m}^3} = .03676 \text{ m}^3$$

$$F_B = \rho_f V_{obj} g = 360 \text{ N}$$

$$F_{grav,block} = 25(g) = 245 \text{ N}$$

$$360 - 245 - m_{extra} g = 0$$

$$m_{extra} = \frac{115}{g} = \boxed{12 \text{ kg}}$$

c) (20 pts) Assume the water tank is open to the air and a hole with a diameter of 2.0 cm is poked in the tank at a level 5.4 meters below the surface. How many gallons per minute flow from the hole? Ignore the block of wood for this part.

$$P_{top} + \rho g y_{top} + \frac{1}{2}\rho V_{top}^2 = P_{bot} + \rho g y_{bot} + \frac{1}{2}\rho V_{bot}^2$$

Assume $P_{top} = P_{bot}$

$$V_{top} = 0$$

$$\rho g (y_{top} - y_{bot}) = \frac{1}{2}\rho V_{bot}^2$$

$$V_{bot} = \sqrt{2gh} = 10.3 \text{ m/s}$$

$$A_{bot} = \frac{\pi(.02)^2}{4} = 3.14 \times 10^{-4}$$

$$q = .003236 \text{ m}^3/\text{s} \cdot \frac{1 \text{ gal}}{3.786 \times 10^{-3} \text{ m}^3} \cdot \frac{60 \text{ s}}{1 \text{ min}}$$

$$= 51.3 \approx \boxed{51 \text{ gal}}$$