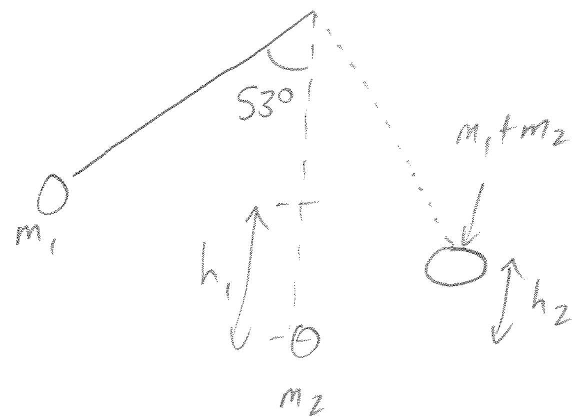
Physics 10154 - Exam #3A

Partial credit will be given provided you show all work and are solving parts of the problem correctly. Points will be deducted if you don't show your work (or if some parts are incorrect) even if you get the right answer. Clearly indicate your answer with a circle or box and remember to include correct units and significant figures.

1. (35 pts) A pendulum bob is attached to a 1.50-meter long string and held at rest with the string making an angle of 53.0° with respect to the vertical. The bob is released and swings toward the lowest point in its motion without any frictional forces. At the lowest point, it strikes a ball of putty, initially at rest, that is twice as massive as the bob, and the two stick together. To what maximum height does the combined mass rise after the collision?



$h_1 = \ell - \ell\cos\theta$

$= 1.50(1 - \cos 53°)$

$= 0.597\,m$

Part 1: m_1 falls

$\Sigma W_F = \Delta K$

$+mgh_1 = \frac{1}{2}m_1 v^2 - 0$ → $v = \sqrt{2gh_1} = 3.42\,m/s$ → v_{2i}

Part 2: collision

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$m(3.42) + 2m(0) = (3m) v_f$

$v_f = 1.14\,m/s$ → v_0

Part 3: $m_1 + m_2$ rise

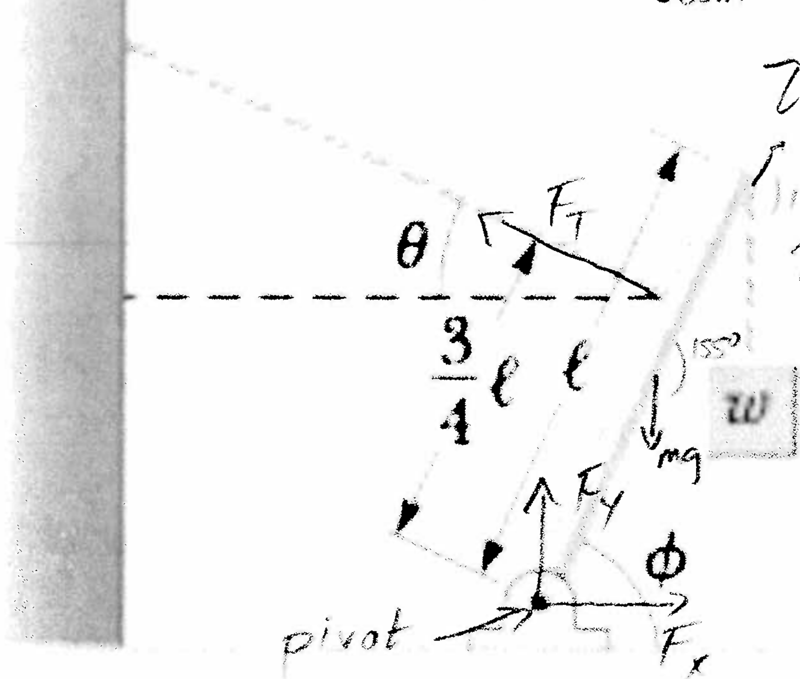
$\Sigma W_F = \Delta K$

$-(m_1 + m_2) g h_2 = 0 - \frac{1}{2}(m_1 + m_2) v_0^2$ → $h_2 = \frac{v_0^2}{2g}$

$= .0664\,m$

2. (35 pts) A 1560-N uniform boom makes an angle Φ = 65.0° above the horizontal. It is supported by a cable that makes an angle θ = 25.0° above the horizontal as shown below. An object with a weight w = 2120 N hangs from the end of the boom. Find (a) the tension in the support cable and (b) the horizontal and vertical components of the reaction force exerted on the base of the boom.

$$\Sigma\tau = \tau_{boom} + \tau_T + \tau_w = 0$$



$$\tau_{boom} = -\left(\frac{\ell}{2}\right) mg \sin 155°$$

$$\tau_T = +\left(\frac{3\ell}{4}\right) F_T \sin 90°$$

$$\tau_w = -(\ell)w \sin 155°$$

$$\Sigma\tau = -\frac{1}{2}\ell(1560)\sin 155° + \frac{3}{4}\ell F_T \sin 90° - \ell(2120)\sin 155° = 0$$

$$= -329.64 + 0.75F_T - 895.95 = 0$$

$$F_T = \frac{1225.59}{0.75} = 1634 \text{ or } 1630 \text{ N}$$

$$\Sigma F_x = F_x - F_T \cos 25° = 0 \qquad F_x = 1634 \cos 25° = 1480 \text{ N}$$

$$\Sigma F_y = F_y + F_T \sin 25° - mg - w = 0$$

$$F_y = 1560 + 2120 - (1630)\sin 25° = 2990 \text{ N}$$

3. (30 pts) Starting from rest, a 12.0-cm radius cylinder rolls without slipping down a 8.50-meter long incline that makes an angle of 17.5° with respect to the horizontal.

a) What is the angular speed of the cylinder at the bottom of the ramp?
b) How many revolutions does the cylinder make during this motion?
c) How long (time) does it take for the cylinder to roll down the ramp?
d) If a 12.0-cm ring were rolled down the ramp instead, how much time would it take?

a) $h = 8.50 \sin 17.5° = 2.556\ m$

$\Sigma W_F = W_{grav} = \Delta K$

$+mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{3}{4}mv^2$

$v = \sqrt{\frac{4}{3}gh} = 5.78\ m/s \qquad \omega = \frac{v}{R} = \boxed{48.1\ rad/s}$

b) $\Delta\theta = \frac{\Delta s}{R} = 70.8\ rad \cdot \frac{1\ rev}{2\pi\ rad} = \boxed{11.3\ rev}$

c) $\Delta\theta = \frac{1}{2}(\omega + \omega_0)t \Rightarrow t = \frac{2(70.8)}{(48.1)} = \boxed{2.94\ s}$

d) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}(MR^2)\left(\frac{v^2}{R^2}\right) = mv^2$

$v = \sqrt{gh} = 5.00\ m/s \qquad \omega = \frac{v}{R} = 41.7\ rad/s$

$t = \frac{2(70.8)}{41.7} = \boxed{3.40\ s}$