Physics 10154 - Summer 2013 - Exam #4A

Instructions: Be sure to SHOW ALL WORK and clearly indicate your answers. I will not give full credit if I cannot logically follow how you got your answer, even if the answer is correct. Partial credit will be given provided you are solving parts of the problem correctly. Clearly indicate your final answer, including correct units and significant figures.

1. (30 pts) A 5.0-kg sphere of radius 6.5 cm is initially at rest at the top of a 2.8 meter long ramp inclined 22° above the horizontal. The sphere rolls without slipping down the ramp. At the bottom of the ramp, determine (a) the sphere's speed, (b) the sphere's angular speed and (c) the sphere's total kinetic energy.

$W_{grav} = \Delta K$

$mgh = \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right) - 0$

$mg\Delta s\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right)$

$(5.0)(9.8)(2.8)\sin 22^\circ = \frac{1}{2}mv^2 + \frac{2}{10}mv^2$

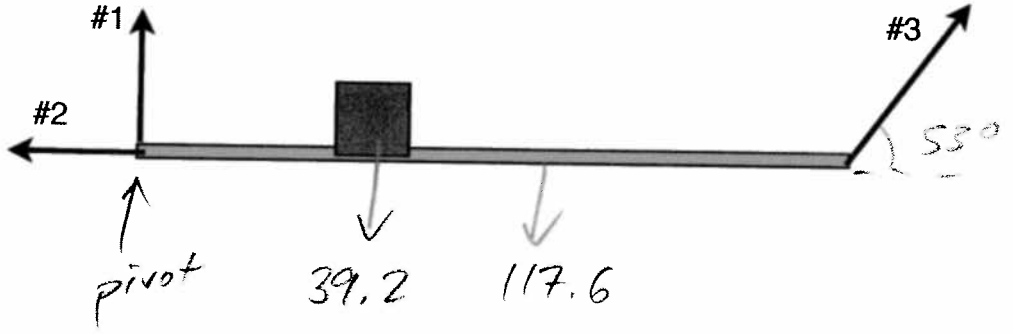
$gh = \frac{7}{10}v^2$

a) $v = \sqrt{\frac{10}{7}(9.8)(2.8)\sin 22^\circ} = 3.8\ m/s$

b) $\omega = \frac{v}{R} = \frac{3.832}{.065} = 59\ rad/s$

c) $K = \frac{7}{10}mv^2 = \frac{7}{10}(5)(3.832)^2 = 51\ J$

2. (30 pts) A thin, uniform 12-kg rod is supported by three ropes as shown below. A 4.0-kg mass rests on the rod 1/3 of the way from the left end. Determine the magnitude of the tension force in each of the three ropes. Rope #1 is vertical, Rope #2 is horizontal, and Rope #3 makes an angle of 53° above the horizontal.



$$\Sigma F_y = F_1 + F_3 \sin 53^\circ - 39.2 - 117.6 = 0$$

$$F_1 + F_3 \sin 53^\circ - 156.8 = 0$$

$$\Sigma F_x = -F_2 + F_3 \cos 53^\circ = 0$$

$$\Sigma \tau = \tau_4 + \tau_{12} + \tau_3 = 0$$

$$\tau_4 = -\frac{\ell}{3}(4.0)(9.8)\sin 90 = -13.07\,\ell$$

$$\tau_{12} = -\frac{\ell}{2}(12)(9.8)\sin 90 = -58.8\,\ell$$

$$\tau_3 = +\ell F_3 \sin 53^\circ = 0.799\,\ell F_3$$

$$-13.07\cancel{\ell} - 58.8\cancel{\ell} + .799\cancel{\ell} F_3 = 0$$

$$0.799 F_3 = 71.87$$

$$F_2 = F_3 \cos 53^\circ \Rightarrow$$

$$F_1 = 156.8 - F_3 \sin 53^\circ \Rightarrow$$

$$\boxed{\begin{aligned} F_3 &= 90\ N \\ F_2 &= 54\ N \\ F_1 &= 85\ N \end{aligned}}$$

3. (40 pts) A large tank of water is open to the air. A small hole in the side of the tank is located 3.5 meters below the water's surface. Water pours out of the hole and into a 5.0 gallon bucket, which fills in 45 seconds. What is the diameter of the hole?

$$(P_{bot} - P_{top}) + \rho g(y_b - y_t) + \tfrac{1}{2}\rho(v_b^2 - v_t^2) = 0$$

same P on top + bottom

$v_{top} \ll v_{bot}$, so $v_{top} \approx 0$ since A_{top} large

$$-\rho g h + \tfrac{1}{2}\rho v_b^2 = 0$$

$$v_b = \sqrt{2gh} = 8.28\ m/s$$

$$A_b v_b = \frac{5.0\ gal}{45\ sec} \cdot \frac{3.786 \times 10^{-3}\ m^3}{1\ gal} = .0004207$$

$$A_b = \frac{.0004207}{8.28} = 5.081 \times 10^{-5}$$

$$\frac{\pi d^2}{4} = 5.081 \times 10^{-5}$$

$$d = 8.04 \times 10^{-3} = 8.0\ mm$$