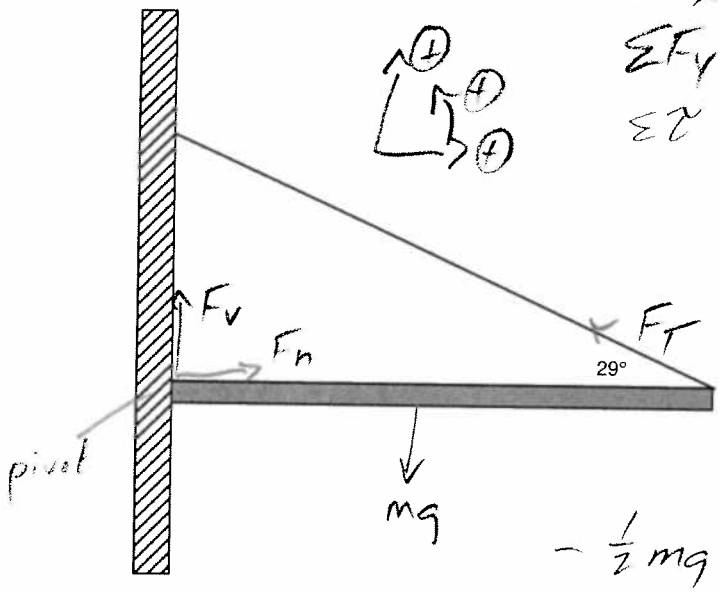
Physics 10154 - Exam #4a

Partial credit will be given provided you show all work and are solving parts of the problem correctly. Points will be deducted if you don't show your work (or if some parts are incorrect) even if you get the right answer. Clearly indicate your answer with a circle or box and remember to include correct units and significant figures.

1. (35 pts) A uniform 45-kg beam of unknown length is attached to a wall and supported in part by a rope attached to the far end. Find the tension in the rope, and the horizontal and vertical components of the reaction force of the wall on the beam.



$\Sigma F_x = F_h - F_T \cos 29° = 0$

$\Sigma F_y = F_V - mg + F_T \sin 29° = 0$

$\Sigma \tau = \tau_{grav} + \tau_T = 0$

$-\frac{1}{2} l\, mg \sin 90$

$+ l F_T \sin 151° = 0$

$-\frac{1}{2} mg + F_T \sin 151° = 0$

$-220.5 + .485 F_T = 0$

$F_T = 454.8\,N$

$\Sigma F_x : F_h = F_T \cos 29° = 397.8\,N$

$\Sigma F_y : F_V = mg - F_T \sin 29° = 220.5\,N$

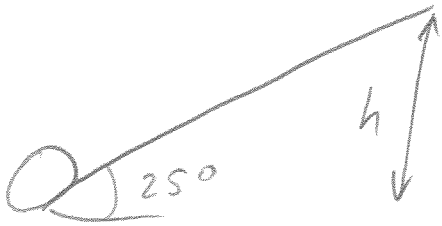
$F_T = 450\,N$

$F_h = 400\,N$

$F_V = 220\,N$

2. (30 pts) A sphere rolls up a 7.5 meter ramp angled 25° above the horizontal with an initial speed of 8.2 m/s. What is the speed of the sphere when it reaches the top of the ramp?

$h = 7.5 \sin 25^\circ = 3.17\,m$



$K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2$

$v_0 = 8.2$

$I = \frac{2}{5}MR^2$

$\omega_0^2 = \frac{v_0^2}{R^2}$

$K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_0^2}{R^2}\right)$

$= \frac{1}{2}mv_0^2 + \frac{2}{10}mv_0^2 = \frac{7}{10}mv_0^2$

$= \frac{7}{10}(m)(8.2)^2 = 47.1m$

$K_f = ?$ at top

$\Sigma W_F = W_{grav} = \Delta K$

$-mgh = K_f - 47.1m$

$-m(9.8)(3.17) = \frac{7}{10}mv^2 - 47.1m$

$-31.1 = \frac{7}{10}v^2 - 47.1$

$22.9 = v^2$

$v = 4.8\ m/s$

3. (35 pts) A container with a wide opening on top is filled with water to a depth of 34 cm. A small circular hole at the bottom of the container allows water to flow out of the hole so that it can fill up a 1.0 gallon container in 15 seconds. The container is surrounded by air on all sides. What is the diameter of the hole? For reference, 1 gallon = 0.003786 m^3.

$P_{TOP} = 101300\ Pa$

$P_{BOT} = 101300\ Pa$

$V_{TOP} = 0$ since $A_{TOP} \gg A_{BOT}$

$Y_{top} = 0.34\ m$

$Y_{bot} = 0$

$$P_{TOP} + \rho g Y_{TOP} + \frac{1}{2}\rho V_{TOP}^2 = P_{BOT} + \rho g Y_{bot} + \frac{1}{2}\rho V_{BOT}^2$$

$$101300 + (1000)(9.8)(.34) + 0 = 101300 + 0 + 500 V_{BOT}^2$$

$$(1000)(9.8)(.34) = 500 V_{BOT}^2$$

$$V_{BOT} = \sqrt{2(9.8)(.34)} = 2.58\ m/s$$

$$Q = A_{BOT} V_{BOT} = \frac{1.0\ gal}{15\ sec} \cdot \frac{.003786\ m^3}{gal} = .0002524\ \frac{m^3}{s}$$

$$\pi r^2 (2.58) = .0002524$$

$$r^2 = 3.114 \times 10^{-5}$$

$$r = .00558$$

$$d = 2r = \boxed{1.1\ cm} \text{ or } .011\ m$$