Physics 10164 - Exam #5a

Show all work. Partial credit will be given provided you show all work and are solving parts of the problems correctly, even if your final answer is incorrect. Points will be deducted if you don't show your work even if you get the right answer. Clearly indicate your answer with a circle or a box (and clearly erase or cross out any work you don't want me to grade) and remember to include the correct units and significant figures in your answer.

1. (30 pts) A certain subatomic particle has a mean lifetime of 0.44 seconds as measured in its own reference frame. An accelerator increases the velocity of the particle such that the mean lifetime is measured to be 2.3 seconds.

a) What is the speed of this particle, expressed as a fraction of the speed of light (e.g. 0.23c).

b) How far does the particle travel during its lifetime, as measured in the rest frame for observers?

c) How far does the particle travel as measured in the particle's own reference frame?

a) $\frac{t_{moving}}{t_{rest}} = \frac{2.3}{0.44} = \gamma$

$\gamma = 5.227 = \frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \Rightarrow 27.3 = \frac{1}{1-\frac{v^2}{c^2}}$

$\Rightarrow 27.3 - \frac{27.3v^2}{c^2} = 1 \quad \frac{v^2}{c^2} = \frac{26.3}{27.3} \Rightarrow \boxed{v = 0.98c}$

b) $d_{moving} = v\, t_{moving}$

$= (.98)(3\times10^8)(2.3) = \boxed{6.8 \times 10^8 \text{ m}}$

c) $d_{rest} = v t_{rest}$

$= (.98)(3\times10^8)(.44) = \boxed{1.3 \times 10^8 \text{ m}}$

2. (30 pts) In a Hydrogen atom, an electron makes four transitions due to photon absorption and emission.

a) From n = 1 to n = 3. — abs
b) From n = 3 to n = 5. — abs
c) From n = 5 to n = 2. — emit
d) From n = 2 to n = 1. — emit

Which of the four transitions represents the highest photon energy emitted by the atom? What is the wavelength of this photon in nm? (d) 10.2 eV, 121 nm

Which of the four transitions represents the ~~highest~~ photon energy absorbed by the atom? What is the wavelength of this photon in nm? (a) 12.1 eV, 103 nm

a) $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$= 1.097\left(1 - \frac{1}{9}\right) \times 10^7 = 9.75 \times 10^6$

$\Delta E = 13.6\left(\frac{8}{9}\right) = 12.1\ eV$ $\quad \lambda = 103\ nm$

b) $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{25}\right) = 1.4 \times 10^6$

$\Delta E = 13.6(.071) = 0.97\ eV$ $\quad \lambda = 713\ nm$

c) $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{4} - \frac{1}{25}\right) = 2.3 \times 10^6$

$\Delta E = 13.6(.21) = 2.86\ eV$ $\quad \lambda = 434\ nm$

d) $\frac{1}{\lambda} = 1.097 \times 10^7\left(1 - \frac{1}{4}\right) = 8.23 \times 10^6$

$\Delta E = 13.6(.75) = 10.2\ eV$ $\quad \lambda = 121\ nm$

3. (40 pts) Plutonium-238 has a half-life of 87.7 years and a mass of 238.049559 u.

a) (10 pts) How much Plutonium-238 (in kg) is required in order for the sample to have an activity of 5.0 Curies?

b) (10 pts) How long (in years) does it take for the activity to drop to a (relatively safe) level of 1.0 microCuries?

a) $T_{1/2} = 87.7 \text{ yrs} = 2.77 \times 10^{9} \text{ s}$

$$\lambda = \frac{.693}{T_{1/2}} = 2.5 \times 10^{-10}$$

$$R = 5.0 \text{ Ci} \cdot \frac{3.7 \times 10^{10} \text{ Bq}}{1 \text{ Ci}} = 18.5 \times 10^{10} \text{ Bq} = \lambda N$$

$$N = \frac{18.5 \times 10^{10}}{2.5 \times 10^{-10}} = 7.4 \times 10^{20} \text{ atoms}$$

$$M = 7.4 \times 10^{20} \text{ atoms} \cdot \frac{238 \text{ u}}{\text{atom}} \cdot \frac{1.66 \times 10^{-27} \text{ kg}}{\text{u}}$$

$$= \boxed{2.9 \times 10^{-4} \text{ kg}}$$

b) $R = R_0 e^{-\lambda t}$

$$1.0 \times 10^{-6} = 5.0 e^{-\lambda t}$$

$$-15.4 = -\lambda t$$

$$t = \frac{-15.4}{2.5 \times 10^{-10}} = 6.17 \times 10^{10} \text{ s}$$

$$= \boxed{1950 \text{ yrs}}$$

c) (20 pts) When Plutonium-238 decays, the result is is often an alpha particle (Helium-4 with a mass of 4.002602 u) and Uranium-234 (mass of 234.040952 u). Determine how much power is generated by a sample of Plutonium-238 with an activity of 5.0 Curies. Answer in Watts.

$\Delta m = {}^{238}Pu - {}^{234}U - {}^{4}He$

$= 238.049559 - 234.040952 - 4.002602$

$= .006005$

$E = 5.59\ MeV$ per reaction

$P = 8.94 \times 10^{-13} \frac{J}{reaction} \cdot 18.5 \times 10^{10} \frac{reactions}{sec}$

$= \boxed{0.165\ Watts}$