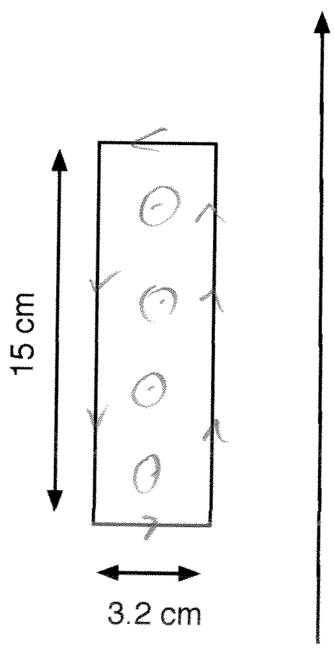
Physics 10164 - Exam 3D

Partial credit will be given provided you show all work and are solving parts of the problem correctly. Points will be deducted if you don't show your work even if you get the right answer. Clearly indicate your answer with a circle or a box and remember to include correct units and significant figures.

1. A long straight wire carries a current of 14.0 Amps. Next to the wire is a rectangular loop. We will assume in this problem that the magnetic field that the loop is immersed in is uniform and that it has a value equal to the magnetic field in the center of the loop due to the long straight wire. The center of the loop is 5.0 cm from the long straight wire. The resistance in the rectangular loop is 0.32 Ohms.

a) (17 pts) Suppose the current in the long straight wire decreases from 14 A to 6.0 A in a time interval of 0.60 seconds. What is the magnitude and direction of the induced current in the rectangular loop? Show your work, including your reasoning for the direction of induced current.



$$B_{loop} = \frac{\mu_0 I}{2\pi r} = \frac{(2\times10^{-7})(14)}{.05}$$

$$= 5.6\times10^{-5}\ T$$

$$B_i = 5.6\times10^{-5}\ T$$

$$B_f = 2.4\times10^{-5}\ T$$

$$\Delta\Phi_B = \Delta B A \cos\theta$$

$$= (3.2\times10^{-5})(.032)(.15)(1)$$

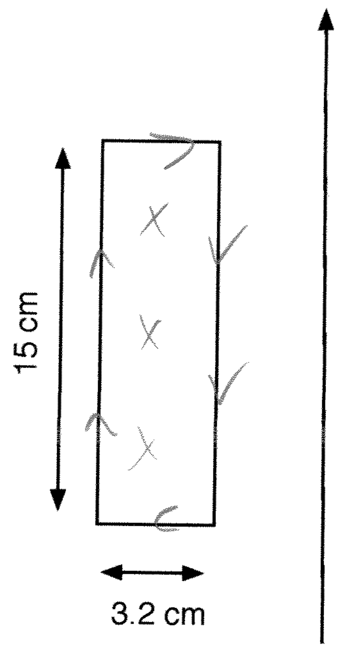
$$= 1.54\times10^{-7}$$

$$\mathcal{E}_{ind} = \frac{\Delta\Phi_B}{\Delta t} = \frac{1.54\times10^{-7}}{.60} = 2.6\times10^{-7}$$

$$I_{ind} = \frac{2.6\times10^{-7}}{0.32} = 8.0\times10^{-7}\ A$$

$\Delta\Phi_B = \odot$, decreasing, $B_{ind} = \odot$, I_{ind} = CCW

b) (16 pts) Suppose the loop moves 2.0 cm closer to the wire in a time interval of 0.45 sec. The current in the long straight wire remains constant for this part at 14 Amps. What is the magnitude and direction of the induced current? Again, show work and reasoning.



$B_i = 5.6 \times 10^{-5}\ T$

$B_f = \dfrac{(2\times10^{-7})(14)}{.03} = 9.33 \times 10^{-5}\ T$

$\Delta\Phi_B = \Delta B A \cos\theta$

$= (3.73 \times 10^{-5})(.032)(.15)(1)$

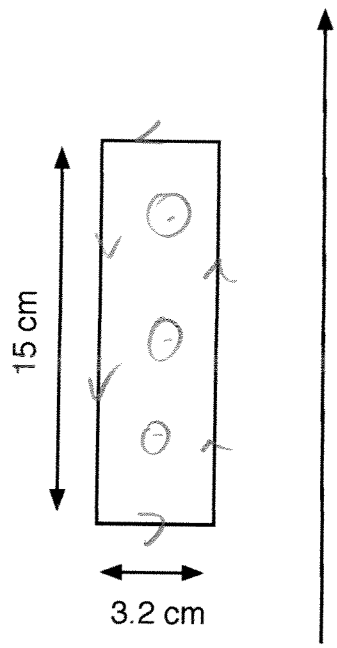
$= 1.8 \times 10^{-7}$

$\mathcal{E}_{ind} = \dfrac{\Delta\Phi_B}{\Delta t} = 4.0 \times 10^{-7}\ V$

$I_{ind} = \dfrac{\mathcal{E}_{ind}}{R} = \dfrac{4.0\times10^{-7}}{.32} = 1.2 \times 10^{-6}\ A$

$\Delta\Phi_B = \odot$, increasing $\Rightarrow B_{ind} = \otimes \Rightarrow I_{ind} = $ CW

c) (17 pts) Suppose the current in the long straight wire is 14 Amps and steady and the loop is back in its original position, with the center 5.0 cm away from the straight wire. The loop then rotates so that the plane of the loop is perpendicular to the page in a time interval of 0.33 seconds. What is the magnitude and direction of the induced current? Again, show work and reasoning.



$B = 5.6 \times 10^{-5}$

$1 - 0$

$\Delta\Phi_B = BA\,\Delta\cos\theta$

$= (5.6\times10^{-5})(.032)(.15)(1)$

$= 2.7\times10^{-7}$

$\mathcal{E}_{ind} = \dfrac{\Delta\Phi_B}{\Delta t} = 8.1\times10^{-7}$

$I_{ind} = \dfrac{8.1\times10^{-7}}{.32} = 2.5\times10^{-6}\,A$

$\Delta\Phi_B = 0$, decreasing $\Rightarrow B_{ind} = 0$, $I_{ind} = $ ccw

2. (30 pts) An AC source operates at a frequency of 60.0 Hz with an rms voltage of 45 Volts. It is connected in series with a 32 Ohm resistor and an unknown inductor. The rms current is measured to be 0.15 Amps.

a) What is the value of the inductance?

b) What is the maximum value of the voltage drop across the resistor?

c) What is the maximum value of the voltage drop across the inductor?

d) When the current is equal to zero in the circuit, what is the voltage drop across the resistor, the inductor and the power source? Briefly explain and/or mathematically justify each of your three answers!

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} \Rightarrow Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \frac{45}{.15} = 300\,\Omega$$

$$300^2 = 32^2 + X_L^2$$

$$\Rightarrow X_L = 298\,\Omega = 2\pi(60)L$$

$$L = 0.79\ H$$

b) $I_{max} = \sqrt{2}\, I_{rms} = 0.212$

$$\Delta V_{R,MAX} = I_{max} R = 6.8\,\Omega$$

c) $\Delta V_{L,MAX} = I_{max} X_L = 63\,\Omega$

d) When $I = 0$, ΔV_R in phase with I is also zero.

$\frac{\Delta I}{\Delta t}$ is maximized, so $\Delta V_L \propto L\frac{\Delta I}{\Delta t}$ is max, $63\,\Omega$

Loop rule: $\Delta V_\varepsilon = \Delta V_R + \Delta V_L = 63\,\Omega$

3. (20 pts) Consider a series RLC circuit with R = 6.5 Ohms, L = 350 mH and C = 120 μF. It has a maximum voltage of 85 Volts.

a) What is the resonant frequency of this circuit?

b) What is the rms value of the current at resonance?

c) If the frequency is reduced to 75% of the resonant frequency, what is the rms value of the current?

a) $f_{res} = \frac{1}{2\pi\sqrt{LC}} = 24.56\ Hz$ or $25\ Hz$

b) $I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$ $\qquad \mathcal{E}_{rms} = \frac{85}{\sqrt{2}} = 60\,V$

At resonance, $Z = R = 6.5\,\Omega$

$I_{rms} = \frac{60}{6.5} = 9.25\ A$ or $9.2\ A$

c) $f_{new} = 18.4\ Hz$

$X_L = 2\pi f L = 40.5\ \Omega$

$X_C = \frac{1}{2\pi f C} = 72.0\ \Omega$

$Z = \sqrt{6.5^2 + (72 - 40.5)^2} = 32\,\Omega$

$I_{rms} = \frac{60}{32} = 1.9\ A$