Physics 10164 - Exam 5A

Partial credit will be given provided you show all work and are solving parts of the problem correctly. Points will be deducted if you don't show your work even if you get the right answer. Clearly indicate your answer with a circle or a box and remember to include correct units and significant figures.

1. (25 pts) An unknown metal has a work function of 3.34 eV, and it is illuminated by light of wavelength 422 nm.

a) Do electrons escape the metal? If no, explain why not. If yes, what is the maximum velocity of escaping electrons?

b) What is the cutoff wavelength of the metal, above which electrons do not escape? Answer in nm.

a) $E_\gamma = \frac{hc}{\lambda} = 4.71 \times 10^{-19}\,J = 2.94\,eV$

Since $E_\gamma < \phi$, electrons cannot escape.

b) Need $E_\gamma = 3.34\,eV = 5.344 \times 10^{-19}\,J = \frac{hc}{\lambda}$

$$\lambda = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{5.344 \times 10^{-19}} = 3.72 \times 10^{-7}\,m$$

or 372 nm

2. (25 pts) A small glowing cloud of Hydrogen gas in a laboratory experiment emits Lyman-alpha radiation (when the electron falls from n = 2 –> n = 1). The total energy emitted at this wavelength is measured to be 7.7×10^{-6} Watts.

a) What is the wavelength (in nm) of the Lyman-alpha radiation?
b) How many photons per second is the gas emitting?

a) $\frac{1}{\lambda} = 1.097 \times 10^{7} \left(\frac{1}{1} - \frac{1}{4} \right) = 8.23 \times 10^{6}$

$\Rightarrow \lambda = 1.21 \times 10^{-7}\ m$

or $\boxed{121\ nm}$

b) $E_{tot} = N_{\gamma} E_{\gamma}$

$E_{\gamma} = 13.6 \left(\frac{1}{1} - \frac{1}{4} \right) = 10.2\ eV$

$= 1.63 \times 10^{-18}\ J$

$N_{\gamma} = \frac{7.7 \times 10^{-6}\ J \text{ (in one second)}}{1.63 \times 10^{-18}\ J}$

$= \boxed{4.7 \times 10^{12} \text{ photons}}$

3. (25 pts) A sample of wood contains 0.030 grams of Carbon-14 as it is measured today. Based on other isotope abundances in the wood, we deduce that it when the tree originally died long ago, the wood contained 3.20 grams of Carbon-14. The half-life of Carbon-14 is 5700 years, and its mass is about 14 amu.

a) What is the activity of the wood today, in Becquerels?
b) How long ago did the tree die, in years?

$$.030 = 3.20\, e^{-\lambda t}$$

$$\lambda = \frac{0.693}{5700} = 1.216 \times 10^{-4}\ yr^{-1}$$

$$\text{or } 3.85 \times 10^{-12}\ s^{-1}$$

$$m_{today} = N_{today}\, m_{C-14}$$

$$.030\ g = N\,(14\ u)\,\frac{1.66 \times 10^{-24}\ g}{u}$$

$$N = 1.29 \times 10^{21}$$

a) $a = \lambda N = (3.85 \times 10^{-12})(1.29 \times 10^{21})$

$$= \boxed{5.0 \times 10^{9}\ Bq}$$

b) $\dfrac{.030}{3.2} = e^{-\lambda t}$

$$-4.67 = (1.216 \times 10^{-4})\, t$$

$$\boxed{t = 37{,}000\ yrs}$$

4. (25 pts) A typical nuclear fission reaction involving Uranium-235 in a nuclear power plant generates 12 MeV of usable energy.

a) What is the binding energy of Uranium-235, in MeV? The atomic number of Uranium is 92. $M_U = 235.043930\,u.$

b) The state of Texas is approximately 28 million people, each of whom averages 5500 Watts of power needed over time (this includes food, clothing, transportation, housing, electricity usage, etc). In the course of a year, how many kg of U-235 would be needed to provide the energy needed for the people in the state of Texas?

a) $N_p = 92$

$N_n = 235 - 92 = 143$

$BE = (92 m_p + 143 m_n - m_U) * 931.5$

$= 1.864557 * 931.5 = \boxed{1740\ MeV.}$

b) $P_{tot} = 1.54 \times 10^{11}\ W$

$E_{tot} = P_{tot} \cdot t = 4.87 \times 10^{18}\ J$

$E_{reac} = 12\ MeV = 1.92 \times 10^{-12}\ J$

$E_{tot} = N_{reac} E_{reac} \Rightarrow N_{reac} = 2.54 \times 10^{30}$

$M_{tot} = N_{tot}\, m_U = (2.54 \times 10^{30})(235 u)\left(\frac{1.66 \times 10^{-27} kg}{u}\right)$

$= \boxed{9.9 \times 10^5\ kg}$