Sp '20 Exam 2A

1. (30 pts) A circuit contains a 12-Volt battery, a 6400 Ohm resistor, and a capacitor, initially uncharged. A switch is closed at time t = 0, closing the loop of the circuit. After 0.23 seconds have elapsed, the capacitor is charged to 55% of its maximum capacity.

a) What is the capacitance of the capacitor?
b) What is the charge on the capacitor at this time?
c) What is the voltage drop across (i) the capacitor and (ii) the resistor at this time?

a) $0.55\,Q_{max} = Q_{max}(1 - e^{-t/RC})$

$0.55 = 1 - e^{-t/RC}$

$0.45 = e^{-t/RC}$

$-0.7985 = -\frac{t}{RC}$

$C = \frac{0.23}{(6400)(.7985)}$

$= \boxed{45\ \mu F}$

b) $Q = C\mathcal{E}(1 - e^{-t/RC})$

$Q_{max} = (45 \times 10^{-6})(12) = 540\,\mu C$

$RC = (6400)(45 \times 10^{-6}) = 0.288$

$Q = 540(1 - e^{-.23/.288})$

$= 540(0.55) = 297\,\mu C$

or $\boxed{3.0 \times 10^{-4}\ C}$

c) $\Delta V_C = \frac{Q}{C} = \frac{3.0 \times 10^{-4}}{45 \times 10^{-6}} = \boxed{6.6\ \text{Volts}}$ (i)

$\Delta V_R + \Delta V_C = 12$ (loop rule) $\Rightarrow \boxed{\Delta V_R = 5.4\ \text{Volts}}$ (ii)

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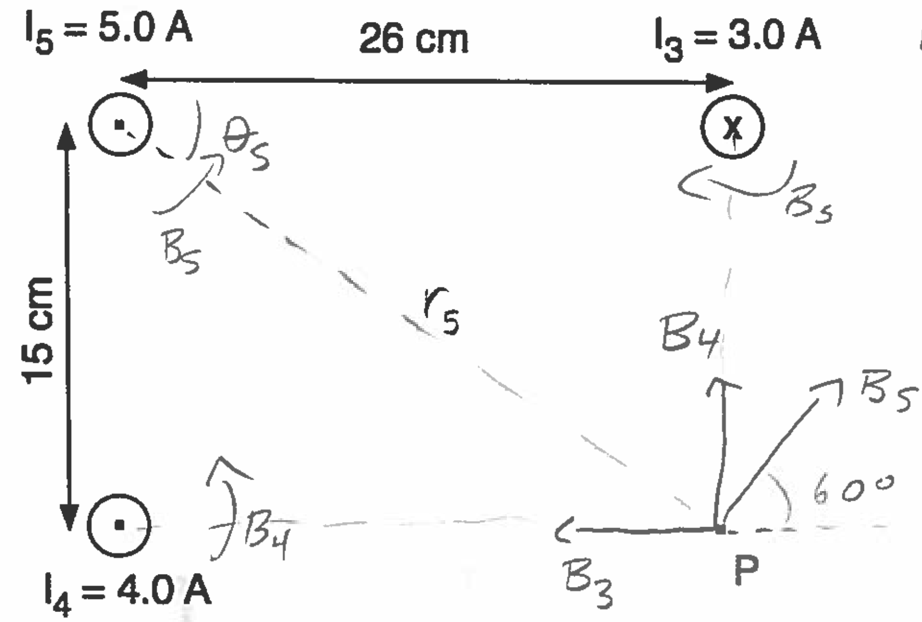
2. (35 pts) Three wires are arranged to form three corners of a rectangle, as shown below. The currents in the wires are all perpendicular to the plane of the rectangle and the page. Point P is in the 4th corner of the rectangle. Find the magnitude and direction of the magnetic field at point P due to the 3 wires.



$r_5 = \sqrt{.26^2 + .15^2} = 0.30\,m$

$\theta_5 = \tan^{-1}\left(\frac{.15}{.26}\right) = 30.0°$

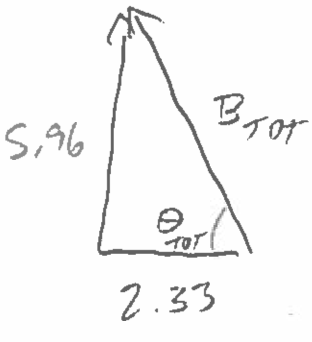
$|B_3| = \frac{\mu_0 (3.0)}{2\pi(.15)} = 4.0\,\mu T$, $-x$ dir $\quad B_{3x} = -4.0 \quad B_{3y} = 0$

$|B_4| = \frac{\mu_0 (4.0)}{2\pi(.26)} = 3.08\,\mu T$, $+y$ dir $\quad B_{4x} = 0 \quad B_{4y} = +3.08$

$|B_5| = \frac{\mu_0 (5.0)}{2\pi(.30)} = 3.33\,\mu T \quad B_{5x} = 3.33\cos 60 = 1.67 \quad B_{5y} = 3.33\sin 60 = 2.88$

60° above $+x$

$B_{TOT} \quad x = -2.33 \quad y = 5.96$



$B_{TOT} = \sqrt{B_x^2 + B_y^2} = $ **6.4 μT**

$\theta_{TOT} = \tan^{-1}\left|\frac{B_y}{B_x}\right| = $ **69° above $-x$**

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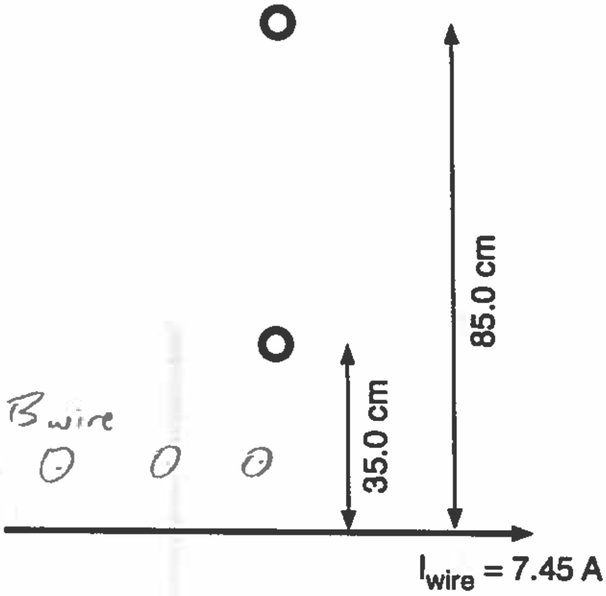
3. (35 pts) A small circular wire loop with 225 turns, a radius of 1.30 cm, and a resistance of 0.540 Ohms is located so that its center is initially 35.0 cm away from a long straight wire carrying a constant current of 7.45 Amps, as shown below.

During a 0.550 second time interval, the loop moves away from the straight wire to a final distance (for the center of the loop) of 85.0 cm. During this time interval, what is the magnitude and direction of the induced current in the loop? You may assume the magnetic field in the loop is constant and has a value equal to whatever its value is at the geometric center of the loop.



$$B_i = \frac{\mu_0 (7.45)}{2\pi (.35)} = 4.26\ \mu T$$

$$B_f = \frac{\mu_0 (7.45)}{2\pi (.85)} = 1.75\ \mu T$$

$$\mathcal{E}_{ind} = \frac{N \Delta B A \cos\theta}{\Delta t}$$

$$= \frac{(225)(4.26 - 1.75)\times 10^{-6}\ \pi (.013)^2 (1)}{.550}$$

$$= 5.45 \times 10^{-7}\ \text{Volts}$$

$$I_{ind} = \mathcal{E}_{ind}/R = \boxed{1.01 \times 10^{-6}\ A}$$

$$\Phi_B = \odot, \downarrow \Rightarrow B_{ind} = \odot$$

$$\Rightarrow \boxed{I_{ind} = \text{ccw}}$$