Sp '20 Exam 2C

1. (30 pts) An empty 28 μF capacitor is connected to a 12 Volt battery and allowed to reach its maximum charge. The capacitor is then disconnected from the battery, and a slab of dielectric material (K = 2.3) is inserted between the plates.

a) What is the new potential difference across the plates of the capacitor?

b) What is the charge on the positive plate of the capacitor after the slab is inserted?

c) If this empty 28 μF capacitor is connected to the 12 Volt battery and in series with a 4.0 Ohm resistor, how long does it take after the circuit is completed for the capacitor to reach 88% of its maximum possible charge?

a) If C disconnected from battery, Q = constant but V can change since $\vec{E}$ is weakened by K.

$$\Delta V_{new} \propto E_{new} \quad \text{so} \quad \Delta V_{new} = \frac{\Delta V_{orig}}{K} = \frac{12}{2.3} = \boxed{5.2\ V}$$

b) $$Q_{orig} = C\Delta V = (28\times10^{-6})(12)$$
$$= 3.36\times10^{-4}\ C$$

same after K inserted, so $\boxed{Q_{new} = 3.4\times10^{-4}\ C}$

c) $$Q(t) = Q_{max}(1 - e^{-t/RC})$$
$$0.88 = 1 - e^{-t/RC}$$
$$0.12 = e^{-t/RC}$$
$$-2.12 = -\frac{t}{(4)(28\times10^{-6})}$$

$$\boxed{t = 2.4\times10^{-4}\ s}$$

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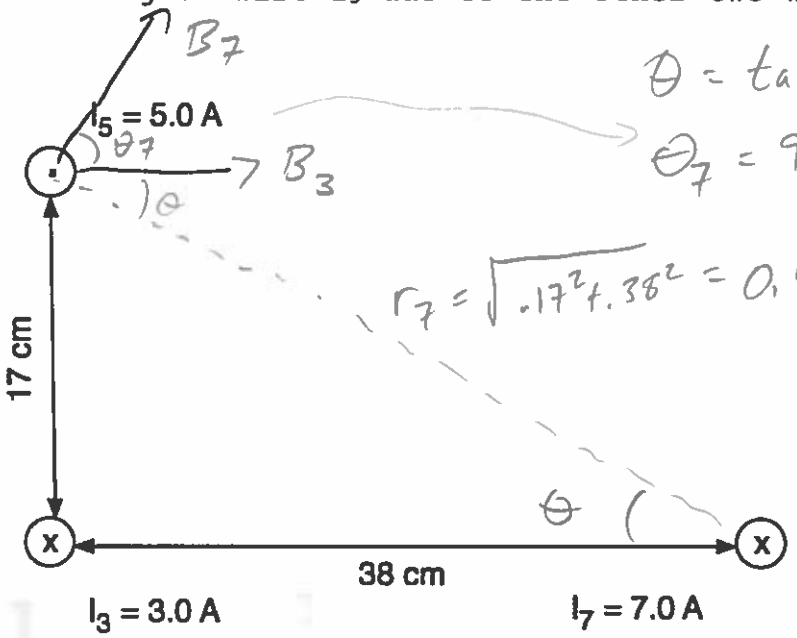
2) (35 pts) Three wires are arranged as shown below.

a) Find the magnitude and direction of the magnetic field at the location of wire I_5 due to the other two wires.

b) Find the magnitude and direction of the force per unit length acting on wire I_5 due to the other two wires.



$\theta = \tan^{-1}\left(\frac{17}{38}\right) = 24.1°$

$\theta_7 = 90 - \theta = 65.9°$

$r_7 = \sqrt{.17^2 + .38^2} = 0.416$

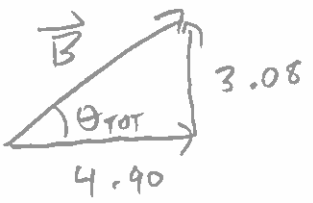
$|B_3| = \frac{\mu_0 I_3}{2\pi r_3} = 3.53\,\mu T$, +x dir

$|B_7| = \frac{\mu_0 I_7}{2\pi r_7} = 3.37\,\mu T$, 65.9° above +x

$B_{3x} = +3.53$ $\quad B_{3y} = 0$

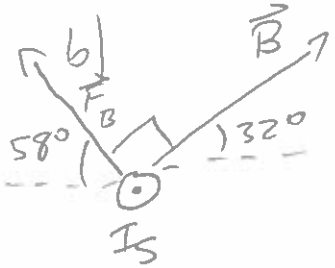
$B_{7x} = +1.37$ $\quad B_{7y} = 3.08$

4.90



$\vec{B}_{TOT} = \sqrt{B_x^2 + B_y^2} = \boxed{5.8\,\mu T}$

$\theta_{TOT} = \tan^{-1}\left|\frac{B_y}{B_x}\right| = \boxed{32° \text{ above } +x}$



$\frac{\vec{F}}{\ell} = (5.0)(5.8\,\mu T)$

$= \boxed{2.9 \times 10^{-5}\,N,\ 58° \text{ above } -x}$

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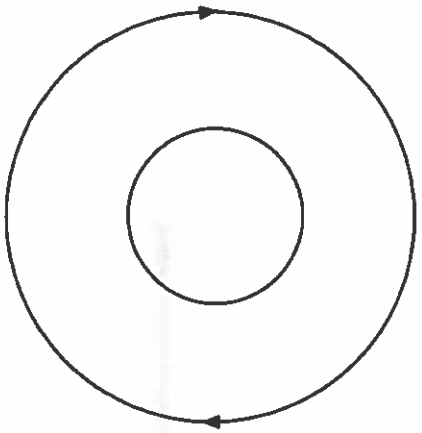
3. (35 pts) Below, we are looking end-on at a 380 turns/cm solenoid of radius 32 cm with a clockwise current of 2.7 Amps. Inside the solenoid is a single-turn wire loop with a radius of 13 cm oriented so that its area vector is parallel with the axis of the solenoid. The current in the solenoid is increased to 9.8 Amps during a time interval of 0.33 seconds.

a) What is the magnitude of the induced EMF in the single-turn wire loop during this time interval?
b) What is the direction of the induced current in the single-turn wire loop during this time interval?
c) What is the magnitude of the torque experienced by the single-turn wire loop during this time interval due to the solenoid's magnetic field?



$B_i = \mu_0 (38000)(2.7) = 0.1293$

$B_f = \mu_0 (38000)(9.8) = 0.4692$

$\Delta B = 0.34\ T$

$$\mathcal{E}_{ind} = \frac{N_{loop} \Delta B A_{loop} \cos\theta}{\Delta t}$$

$$= \frac{(1)(0.34)\pi(.13)^2(1)}{.33} = \boxed{.055\ V}$$

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b) $\Delta \vec{B} = \otimes$, increasing

$\rightarrow B_{ind} = \odot$

$\Rightarrow \boxed{I_{ind} = ccw}$

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c) Since $\vec{\mu}_{loop}$ & $\vec{B}_{ext}$ are parallel, $\sin\theta = 0$

so $\boxed{\vec{\tau} = 0}$