Physics 10263 Lab #6: Cepheid Variables

Introduction

In a previous lab, we used the inverse square law to determine the distance to the Pleiades cluster. The inverse square law is one of the most common distance indicators used in astronomy. As a reminder, it can be written as follows:

\[ \frac{100}{r} = \frac{L_{\text{app}}}{L_{\text{abs}}} - X \]

This is the “linear form” of the inverse square law. Assuming you can measure the apparent luminosity of an object, the “X” correction term for interstellar extinction and redding, you need to find some way to measure the absolute luminosity. If you are successful in these three tasks, you can solve the equation above for the distance (r).

Another form of the inverse square law is often used by astronomers. It uses the logarithmic form of the luminosity (apparent and absolute magnitudes):

\[ m = \text{apparent magnitude} = -2.5 \log L_{\text{app}} \]
\[ M = \text{absolute magnitude} = -2.5 \log L_{\text{abs}} \]
Substituting these terms into the inverse square law and solving for \( r \), we get:

\[
\frac{\left( m - M + 5 - x \right)}{5} = 10
\]

\( r \) (pc) = distance to the star (in parsecs)
\( m = \) apparent magnitude of the star
\( M = \) absolute magnitude of the star
\( x = \) correction term for extinction

When finding the distance to the Pleiades cluster, we used the spectral type information for individual stars in the cluster. That information translated into an estimate of the absolute luminosity. Combined with the measured apparent luminosity and an estimate of "\( X \)", we were able to find the distance to that cluster.

Finding the distance to other galaxies, however, can be a bit more difficult. Whereas the Pleiades cluster is about 120 parsecs away, the Small Magellanic Cloud (a small satellite galaxy orbiting our own Milky Way) is about 53,000 parsecs away, making observations of individual stars much more difficult. Thus, we cannot use the Pleiades method with the Small Magellanic Cloud since the only individual stars we can successfully pick out are the rare, very bright ones.

Fortunately, some of these very bright stars belong to a class of stars called **Cepheid Variables**. Cepheid Variables are typically anywhere from 100 to 10,000 times more luminous than our own Sun, and their brightnesses vary with periods from 1 to 100 days. The prototype of this group is the fourth brightest star in the constellation Cepheus, Delta Cephei, the properties of which were discovered in 1784 by John Goodricke, an English amateur astronomer. After more such stars were discovered, theorists surmised that the brightness variations were caused by pulsations in the stars (growing larger and smaller periodically).
In 1912, Harvard astronomer Henrietta Leavitt discovered a correlation between the absolute luminosity and the period of Cepheid Variable stars. The longer the period of the star, the higher its absolute luminosity (this proportionality is known as The Cepheid Period-Luminosity Relation). This proved enormously beneficial to astronomers, since the variation period in a light curve of a star is much easier to determine than the spectral type (the light curve period is also a much more accurate indicator of absolute luminosity, as it turns out).

In this lab, we will study four Cepheid Variable stars located in the Small Magellanic Cloud (SMC). Even if these stars are spread out all over the SMC, we can assume they are all at the same distance. Assuming the distance to the SMC is 53,000 parsecs, the worst case would be that one would be at, say, 52,000 parsecs and another would be at 54,000 parsecs. In other words, our experimental error in determining the distance to the SMC (probably around 10% - 20% of the distance) will likely be larger than the size of the cloud itself (probably only 1% - 2% of the distance).

Procedure

Before we can use the Cepheid Period-Luminosity (P-L) Relation to determine the distance to the SMC, we need to calibrate it. In other words, we need to determine the actual relationship between period and absolute luminosity by using a sample of local stars (whose absolute luminosities we can determine independently). Once we have this information, we’ll find the periods of stars in the SMC, use the P-L relation to guess the absolute luminosity, and thus find the distance.
Fortunately for us, two astronomers (Walter Baade and Robert Kraft) have already calibrated the P-L relation for us. Their work is summarized in the table that follows.

<table>
<thead>
<tr>
<th>log P</th>
<th>M (Baade)</th>
<th>M (Kraft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>-3.7</td>
<td>-3.4</td>
</tr>
<tr>
<td>1.00</td>
<td>-4.4</td>
<td>-3.8</td>
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<tr>
<td>1.20</td>
<td>-5.1</td>
<td>-4.8</td>
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<tr>
<td>1.80</td>
<td>-7.3</td>
<td>-7.0</td>
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</table>

(1) On your worksheet graph, plot the six points from the table \( \log P \) vs \( M \) (Baade). Note that the scale for Absolute Magnitude \( M \) is on the lower half of the graph. To save space, part of the \( y \)-axis has been cut out. After you have plotted the six points, use a ruler and draw the best fit straight line that passes as close as possible (but not necessarily through) all six points. Label this line “M(Baade)”.

(2) On the same graph, plot the six points from the table \( \log P \) vs \( M \) (Kraft). Again, use the lower half of the graph since you’re plotting absolute magnitudes. After you have plotted the six points, use a ruler and draw the best fit straight line that passes as close as possible (but not necessarily through) all six points. Label this line “M(Kraft)”.

We have now plotted the calibrated P-L relation (two versions) for all Cepheid Variables. Next we will plot the period and apparent magnitude information for Cepheids in the SMC. This information, combined with the information in the calibrated graph, will provide us with an estimate of the distance to the SMC.
(3) On the data sheet that accompanies these notes are light curves of four Cepheid Variables based on photographic observations by astronomer Halton C. Arp. Estimate to a precision of 0.1 magnitudes the average value of the apparent magnitude of the minima of each light curve (note the magnitude scales on the right and left are identical and the increase downward). Enter this data in column 1 of the table on your worksheet.

(4) For each star, determine the time between any two minima (for greater accuracy, measure across more than one minima and divide by the number of periods you’ve covered) to a precision of 1 day. Enter this data in column 2 of the table on your worksheet.

(5) Take the logarithm of each of your four period values in column 2. Write these values in column 3 of your table with three significant figures.

(6) Now, on the upper part of the graph on your worksheet, plot the data from column 3 (log P) vs the data from column 1 (apparent magnitude). As in parts (1) and (2), after you’ve plotted your four points, draw the best fit straight line that passes as close as possible (but not necessarily through) all four points. Label this line “m”.

You should now have three roughly parallel lines on your worksheet graph, one at the top and two at the bottom.

(7) For each of the four points on your top (apparent magnitude) graph, use a ruler to draw a vertical line straight down from that point all the way to the bottom of the graph. These four lines will intersect your other two absolute magnitude graphs.

Using these lines as guides, read off the value (to a precision of 0.1 magnitudes) of the M(Baade) line where the M(Baade) graph intercepts your four vertical lines. Enter this data into column 4 of your table. For each of the four stars you measured, you now have a value of apparent and absolute magnitude!

(8) Again using the lines as guides, read off the value (to a precision of 0.1 magnitudes) of the M(Kraft) line where the M(Kraft) graph intercepts your four vertical lines. Enter this
data into column 5 of your table. Now we have two absolute magnitude estimates for each of the four stars. We’ll determine which astronomer did a better job calibrating the data!

(9) In column 6 of your table, calculate “m - M(Baade)”. In other words, subtract column 4 from column 1. Your answer should have a precision of 0.1 mag.

(10) In column 7 of your table, calculate “m - M(Kraft)”. In other words, subtract column 5 from column 1. Your answer should have a precision of 0.1 mag.
(1) - (2) See back of this page for graph.

(3) - (10)

<table>
<thead>
<tr>
<th>Star</th>
<th>Apparent Magnitude (m)</th>
<th>Period (days)</th>
<th>Log P</th>
<th>M (Baade)</th>
<th>M (Kraft)</th>
<th>m - M(Baade)</th>
<th>m - M(Kraft)</th>
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<tbody>
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</table>

(11) Average value of "m - M(Baade)" = ______________
Average value of "m - M(Kraft)" = ______________

(12) Distance to the SMC (Baade’s calibration): _____________ pc
% error in Baade’s calibration: _____________ %

Distance to the SMC (Kraft’s calibration): _____________ pc
% error in Kraft’s calibration: _____________ %
No essay for this week’s lab.