## Physics 10263 Lab \#9: <br> Measuring the Hubble Constant

## Introduction

In the 1920's, Edwin Hubble discovered a relationship that is now known as Hubble's Law. It states that the recession velocity of a galaxy is proportional to its distance from us. The equation used to express this is:
$V=H * d$, where

$$
\begin{aligned}
& \mathrm{V}=\text { recession velocity of the galaxy (in km/sec) } \\
& \mathrm{H}=\text { Hubble constant (in } \mathrm{km} / \mathrm{sec} / \mathrm{Mpc} \text { ) } \\
& \mathrm{d}=\text { distance to galaxy (in Mpc, or millions of parsecs) }
\end{aligned}
$$

This equation tells us that a galaxy moving away from us twice as fast as another galaxy will be twice as far away.

The velocity is relatively easy for us to measure using the Doppler effect. An object in motion will have its radiation (i.e. light) shifted in wavelength according to the following formula:

$$
\begin{aligned}
& \mathrm{V} * \lambda_{\text {rest }}=\mathrm{C} *\left(\lambda-\lambda_{\text {rest }}\right), \text { where } \\
& \left.\lambda=\text { observed wavelength (in Angstroms, or } 10^{-10} \mathrm{~m}\right) . \\
& \lambda_{\text {rest }}=\text { rest wavelength } \\
& \mathrm{V}=\text { speed of the object }(\text { in } \mathrm{km} / \mathrm{sec}) \\
& \mathrm{C}=\text { the speed of light }\left(3.0 \times 10^{5} \mathrm{~km} / \mathrm{sec}\right)
\end{aligned}
$$

So, we can determine the velocity of a galaxy from its spectrum. We simply measure the wavelength shift (the difference between observed and intrinsic wavelength) of a known spectral absorption line and solve for $v$.

Example: Suppose a certain absorption line has a true wavelength of 5000 Angstroms. When analyzing the spectrum of a particular galaxy, we observe the absorption line at a wavelength of 5050 Angstroms. We then conclude that the galaxy is moving away from us with a velocity of:

$$
\mathrm{v}=[(5050-5000) / 5000] * 300,000=3000 \mathrm{~km} / \mathrm{sec}
$$

In order to estimate the distances to galaxies, we're going to use a distance determination technique known as the "standard ruler". In this technique, we assume that all galaxies have the same intrinsic linear size (just like all rulers are $12^{\prime \prime}$ long). In other words, if one says that the Milky Way is 120,000 light years (about 37,000 parsecs, or 37 kiloparsecs) across, then we would assume that every galaxy in the sky also measures 120,000 light years across.

We can now use a method similar to parallax to determine the distances to the galaxies in our data set.


In the diagram above, we are assuming that we know s (the standard ruler size of 37 kiloparsecs, or kpc ), and we wish to measure A in order to find d. To determine the distance, d, to a galaxy with a known linear size (s) and known angular size, we simply use the small-angle formula:
$d=S / A, \quad$ where
$\mathrm{d}=$ distance to the galaxy (in Mpc)
$\mathbf{S}=$ linear size of the galaxy (in kpc)
$A=$ angular size of the galaxy (in milliradians)
You can measure the angular size of the galaxy, $A$, in milliradians, provided you know the proper scaling factor. In this case, the scale factor for converting a measurement in millimeters into an angular size in milliradians is 0.037 mrad/ mm.

Example: Suppose we measure the photographic size of the galaxy to be 14 mm . To convert this measurement into an angular size, simply multiply by the scale factor:

$$
A=14 \mathrm{~mm} * 0.037 \mathrm{mrad} / 1 \mathrm{~mm}=0.52 \mathrm{mrad}
$$

For this lab, we are going to assume that the standard linear size for a galaxy is 22 kpc (or about 70,000 light years, which is small compared to the Milky Way, but we know that the Milky Way is unusually large compared to most galaxies).

## Step 1

There are 19 different galaxy data sheets in your lab manual after the text of this lab. Each data sheet has the name of the galaxy, a photograph of the galaxy and a partial spectrum of the galaxy, showing the intrinsic and observed wavelengths of two different absorption lines (the Calcium "H" and "K" lines). For 12 different galaxies, fill in the data table (which is a few pages ahead of this text) as follows:
(1) - Fill in the galaxy catalog number
(2) - Measure the size of the galaxy on paper in millimeters with two significant figures. If the galaxy is not circular in shape, measure the longest axis.
(3) - Use the scale factor ( $0.037 \mathrm{mrad} / \mathrm{mm}$ ) as above to convert the measured size to an angular size.
(4) - Assuming the linear size of the galaxy is 22 kpc , use the standard ruler equation above ( $d=s / A$ ) to calculate the distance to the galaxy in Mpc. Write down this distance with two SF.
(5) - Measure the location of the center of the Ca-K absorption line (the left of the pair) to the nearest Angstrom.
(6) - Subtract the true value (3933 Angstroms) of the Ca-K wavelength from the measured value of the Ca-K wavelength (should get a positive number). This is the "Ca-K shift".
(7) - Measure the location of the center of the Ca-H absorption line (the right of the pair) to the nearest Angstrom.
(8) - Subtract the true value (3969 Angstroms) of the Ca-H wavelength from the measured value of the $\mathrm{Ca}-\mathrm{H}$ wavelength (should get a positive number). This is the "Ca-H shift".
(9) - Average your values from column (6) and column (8) to get the average measured redshift.
(10) - Using the wavelength of the Ca-K line as your "rest" wavelength ( 3933 Angstroms), use the equation on page 1 to calculate the velocity of this galaxy. Write down this velocity with two SF.

One of the galaxies (NGC 3147) has already had its properties calculated for you. Double-check the entries in the table yourself so that you know how to fill out the rest of the table for other galaxies with confidence! In making your measurements, you were bound to come across some uncertainties. For example, suppose that you conclude the photographic size of a particular galaxy is 10 mm , but your neighbor decides to measure further out into the faint fringes and measures an angular size of 20 mm .

Q1. Will your estimated distance to that galaxy be larger or smaller? Explain your answer below.

## Step 2

On the graph paper provided, plot your values for distance (Mpc) and velocity (km/sec). Draw a straight line that passes through the origin and best fits the general slope of the points on the graph. Use a ruler to ensure the line is straight. Before committing to drawing your line, move your ruler around to gauge for yourself what is the best fit to these data points.

Measure the slope of your line, (rise / run) or (change in v / change in d). This is the Hubble constant, H.

Q2. What is the slope of your line? $\qquad$ km/sec/Mpc

## Step 3

It turns out that the Hubble constant is a measure of the age of the Universe. To determine roughly what the age of the Universe is from your Hubble constant, perform the following calculation. We'll discuss this calculation much more during lecture.

Age (in billions of years) $=700 / \mathrm{H}$. (use two SF)
Q3. Estimated age of Universe: $\qquad$ billion years old.

## Step 4

Suppose you systematically underestimated the photographic sizes of the galaxies, as discussed in question \#2.

Q4a. Would your slope be larger or smaller than the "true" slope? Explain!

Q4b. Will your age for the Universe be smaller or larger than the "true" age of the Universe? Explain!

## Step 5

Suppose we are wrong about the typical linear size of a galaxy. Instead of 22 kpc , it is really more like 30 kpc (closer to the size of the Milky Way). We decide to recalculate everything using this new value.

Q5. Will our estimated age for the Universe be larger or smaller? Explain!

## Step 6

Suppose new observations were reported tomorrow in which astronomers are arguing for the existence of a star with a very low mass and an observed age of 100 billion years.

Q6. Would this necessarily conflict with your results? Explain!

## Essay

No essay for this lab. Answers to steps 1-6 are sufficient analysis and interpretation.

## Credits

The galaxy spectra for this lab were obtained from Robert C. Kennicutt, Jr. of the University of Arizona. They are published in the Astrophysical Journal Supplement Series, volume 79, pp. 255-284, 1992. The digital images of the galaxies have been extracted from the Palomar Digital Sky Survey, which can also be found on the World Wide Web at http://stdatu.stsci.edu/cgi-bin/ dss_form. We gratefully acknowledge the various copyrights for this work, information about which is available at the listed URL.

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The Hubble Relation


Lab \#9 Data


NGC 1832


NGC 2276


NGC 2775


NGC 2903


NGC 3034


NGC 3147


NGC 3227


NGC 3245


NGC 3368


NGC 3516


NGC 3623


NGC 3627


NGC 3941


NGC 4775


NGC 5248


NGC 6181


NGC 6217


NGC 6643


