Astronomy 101U Homework #3 – Stars

Answer the following questions on your own paper. Please show all work and answer each question completely. Questions are worth 10 points each.

It was once thought that the source of the Sun's energy was gravitational contraction, a process that we know generates heat even today (that's how protostars initially ignite their nuclear fuel). We know that the Sun must generate energy as fast as it releases it. The total energy released by the Sun per second is about 4 x 10^{33} ergs. So if we can just find out how much energy the Sun can release over its whole lifetime (in ergs), we can determine how long it will live.

A similar process can be used for a car. Say a car uses up 2 gallons of gas per hour. If we figure out how much fuel the car can hold (say, 20 gallons), then we know the car's lifetime of burning 2 gallons of fuel every hour will be 10 hours.

(1) If we assume the Sun gets all of its energy from gravitational contraction, we find that the Sun has 4 x 10^{48} ergs of available "fuel". How long can the Sun live (in years) with gravity as its only source of energy?

This result was widely known and accepted around the turn of the century, until geologists using radioactive dating determined that the age of the Earth was about 5 x 10^9 years (5 billion years) old, much older than the apparent age for the Sun estimated in (1)! In fact, this was used as a creationist argument, supporting the fact that the Earth is indeed the oldest object in the Universe and hence must have been created.

Much later, when the theory of nuclear fusion was being developed, scientists applied their theory to stellar energy generation and the Sun. Einstein's famous $E = mc^2$ equation doesn't quite work here, since the reaction involved only converts a small fraction of the mass involved into useful energy. Still, there's a lot of energy involved.

(2) Assuming that 10% of the Sun's mass is available for nuclear energy generation during its main sequence lifetime (only the Hydrogen close to the core, for reasons we will get into in class), and assuming that the mass is converted into energy with an efficiency of only 0.7% ($E = 0.1*0.007*mc^2$), calculate the total energy (in ergs) of the Sun. Note that the mass of the Sun is 2 x 10^{33} grams and the speed of light is 3 x 10^{10} cm/sec.

(3) Now estimate the lifetime of the Sun in years as in (1) with nuclear fusion as the power source. Your answer should be more in line (in order of magnitude at least) with the estimated age of the Earth.

(4) We said in problem (2) that only the Hydrogen close to the core of the Sun or any other star will participate in the nuclear energy generation of the star during its main sequence lifetime. Explain why this is true.

Now we'll have a look at stellar evolution and some of the reasons behind the phenomena we see in the sky.

(5) Massive stars have much more "fuel" than stars like the Sun, so why do they have such a short lifetime? Hint: How does the rate of nuclear reactions relate to temperature (which in turn relates to mass)?

(6) What is the mechanism that we think causes red giants to be so enormous? In other words, what is going on inside the star that causes the envelope to puff out so much?

(7) Why does it require higher and higher temperatures in the star's core in order to fuse heavier and heavier elements?

Earlier in this century, a famous astronomer named Subrahmanyan Chandrasekhar determined that stars of a certain mass (known as the Chandrasekhar limit) would be so massive that the escape velocity from their surface would exceed the speed of light. Objects of this type eventually came to be known as black holes since not even light could escape from them.

(8) The escape velocity can be found from the equation $v^2 = GM/R$ where G is the gravitational constant (6.7 x 10^{-8} in units of $cm^3/gm\text{-}sec^2$), R is the radius of the black hole (about 2 x 10^5 cm or 2 km...use cm in your calculations to keep all your units straight), and M is the mass of the star. Plug in the speed of light, c, for the escape velocity and solve for the mass of the star (in grams). Convert your answer into solar masses.

(9) Your answer for (8) should be fairly close to the mass of the Sun, but the text says that only stars with an initial mass of about 6 solar masses ever reach the stage in which they can become black holes. Why is this?

One of a scientists' favorite activities to stretch the mind and kill time is the so-called "back of the envelope" calculation. Let's try one to see if we can get a handle on the energy released in a supernova explosion.

(10) When it first ignites, a rough estimate for the luminosity of a supernova is 10^{51} ergs/sec (about 10^{18} or a billion billion times more luminous than the Sun). We are located at about 10^{13} cm from the Sun. Let's imagine a supernova goes off there. We want to compare the amount of energy we receive from a supernova explosion to the energy we receive (remember the inverse square law...it goes as $1/r^2$) at a distance of about 1 km from an atomic blast (1 km = 10^5 cm). A modern atomic weapon is rated at 50 Megatons (that's the amount of energy it can release), which is roughly 10^{24} ergs released per second. What is the ratio of the energy received from a supernova explosion on the Earth from the Sun to the ratio of energy we'd receive viewing a nuclear explosion from 1 km away?