# Experiment 

# 6 

## AC Circuits and Electrical Resonance



Fig. 1. The series combination.
Consider an AC circuit containing a resistor, an inductor, and a capacitor connected in series, as seen in Fig. 1. Please note that the same current flows through all three elements. Since the current is common to all elements, we will take it as a reference, and will measure voltages across the resistor, the capacitor and the inductor with respect to the current. It is convenient to present the results in the form of a graph in which the horizontal axis represents the current, see Fig. 2.

The voltage across the resistor is given by Ohm's law
$\mathrm{V}_{\mathrm{R}}=\mathrm{IR}$
and is in phase with the current. Thus, $\mathrm{V}_{\mathrm{R}}$ is displayed on the x -axis.
The voltage across the inductor,
$\mathrm{V}_{\mathrm{L}}=\omega \mathrm{LI}$
leads the current by $90^{\circ}$, and it will be presented along the $y$-axis.
The voltage across the capacitor,
$\mathrm{V}_{\mathrm{C}}=\mathrm{I} / \omega \mathrm{C}$


Fig. 2. Phase diagram for a series RLC combination.
lags the current by $90^{\circ}$, and is also presented on the vertical axis.
To obtain the resultant voltage, $\varepsilon$, we need to add voltages $\mathrm{V}_{\mathrm{R}}, \mathrm{V}_{\mathrm{L}}$, and $\mathrm{V}_{\mathrm{C}}$ as
vectors. The vector addition is illustrated in Fig. 2. Because the vectors $V_{R}$ and $V_{C}$ or $V_{L}$ form a right triangle, $\varepsilon$ may be found from

$$
\begin{equation*}
\varepsilon^{2}=\mathrm{V}_{\mathrm{R}}^{2}+\left(\mathrm{V}_{\mathrm{L}}-\mathrm{V}_{\mathrm{C}}\right)^{2} \tag{4}
\end{equation*}
$$

Substituting Eqs. 1-3 in Eq. 4, leads to

$$
\begin{equation*}
\varepsilon=\mathrm{I}\left[\mathrm{R}^{2}+(\omega \mathrm{L}-1 / \omega \mathrm{C})^{2}\right]^{1 / 2}=\mathrm{I} \mathrm{Z} \tag{5}
\end{equation*}
$$

where $\mathrm{Z}=\left[\mathrm{R}^{2}+(\omega \mathrm{L}-1 / \omega \mathrm{C})^{2}\right]^{1 / 2}$, and is called the impedance of the circuit. The phase difference between the current, I, and the line voltage, $\varepsilon$, is given by
$\tan \phi=(\omega \mathrm{L}-1 / \omega \mathrm{C}) / \mathrm{R}$
From Eq. 6 it is seen that for $\omega \mathrm{L}=1 / \omega \mathrm{C}$ the phase difference, $\phi$, is zero and the impedence, $Z$, equals $R$. This condition

$$
\begin{equation*}
\omega \mathrm{L}=1 / \omega \mathrm{C} \tag{7}
\end{equation*}
$$

is called resonance, and it can be reached by changing any of the quantities $\omega, \mathrm{C}$, or L . In this experiment, to reach resonance, you will change only C, while $\omega$ and L remain fixed. When the Z equals R , the current is maximum and the circuit is said to be in resonance.

In this experiment you will find the resonance by changing the capacitance of the system. The frequency $\omega$ is determined by the power supply; in this case it is $120 \pi / \mathrm{s}$. $\omega$ measured in radians per second, and is related to the frequency $\mathrm{f}=60 \mathrm{~Hz}$, measured in cycles per second as $\omega=2 \pi \mathrm{f}$.

## 2. Procedure

The RLC circuit was built and installed inside a box with a transparent plastic cover so you can see all the connections inside, compare Fig. 3. In addition there are
several binding posts, which enable voltage measurements between different elements. Do not touch metal parts of the posts with your hands. To measure the voltage use special leads provided.


Fig. 3. Experimental setup with an ammeter attached.

## 1. DISCONNECT TBE POWER CORD

2. Set the selector switch to the S position.
3. Insert the plug in $\mathbf{J} \mathbf{2}$ and connect the leads to the ammeter, as in Fig. 3. Set switch on the ammeter to the 50 milliampere position.
4. Plug in the power cord. Measure and record the current on the ammeter, the line voltage, $\varepsilon$, between posts $\mathbf{P}$ 2 and $\mathbf{P} 3$, the voltage across the capacitors, $\mathrm{V}_{\mathrm{C}}$, across $\mathbf{P} 1$ and $\mathbf{P}$ 2, and the voltage across the inductance, $\mathrm{V}_{\mathrm{L}}$, between $\mathbf{P} 1$ and $\mathbf{P} 3$, for various values of capacitance. By closing the switches you can get different values of resultant capacitance. The capacitors are connected in parallel and to find the resultant capacitance just add the values of individual capacitances printed on the box. Start from zero and increase C in steps of $0.1 \mu \mathrm{~F}$. You will observe that with increasing C the current increases, reaches the maximum, and then decreases. Near the maximum change the step to $0.05 \mu \mathrm{~F}$ and even to $0.02 \mu \mathrm{~F}$. These additional points will enable better characteristics of the maximum, and thus the resonance conditions. Graph the current, I, the line voltage, $\varepsilon$, the voltage across the capacitor,
$\mathrm{V}_{\mathrm{C}}$, and the voltage across the inductor $\mathrm{V}_{\mathrm{L}}$ as a function of capacitance, C , (on the horizontal axis). You will need 10 to 20 points to make good quality graphs.
5. DISCONNECT THE POWER CORD. Remove the plug from J2 and set the selector switch to the S position. With the digital meter set to ohms, measure the resistance of the inductor, $\mathbf{P 1}$ and $\mathbf{P 3}$. You measured directly the ohmic resistance of the inductor, $\mathrm{R}_{\mathrm{L}}$. In addition, there is a resistor R connected in series. The resultant ohmic resistance of the circuit is the sum of both values, $\mathrm{R}_{\mathrm{eq}}=\mathrm{R}+\mathrm{R}_{\mathrm{L}}$. Calculate the current I from $\mathrm{V} / \mathrm{R}_{\text {eq. }}$. In the report, in the discussion section, explain why that value was different from that obtained by direct current measurement.

## 3. Report

In the introduction discuss applications of the series combinations of $\mathrm{R}, \mathrm{L}$ and In this experiment graphs are very important, and all conclusions should be based on the analysis of the graphs. According to Eq. 7 maximum current results when $\omega \mathrm{L}=$ $1 / \omega \mathrm{C}$. We will call the value of C when the resonance takes place the resonance capacitance and denote it as $C_{\text {res }}$. Remember that $\omega=2 \pi \mathrm{f}$. Using $\mathrm{f}=60$ cycles per second, and the value of $\mathrm{C}_{\text {res }}$ determined from the graphs, calculate inductance L .

