Experiment
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Spectrum of Hydrogen and Rydberg Constant

1.   Introduction

The electrons in an incandescent light source undergo thermal excitation and emit electromagnetic radiation (light) of many different wavelengths, producing a continuous spectrum. However, when light emitted from excited gases or vaporized liquids, or solids is analyzed, line spectra are observed. Different substances have characteristic spectra -- i.e. they have a characteristic set of lines at specific wavelengths. The characteristic color of light from a gas discharge is indicative of the most intense spectral line or lines in the visible region. For example, light from a hydrogen discharge has a red glow due to an intense emission line with a wavelength of 656.1 nm.

The characteristic spacing of the spectral lines in the spectrum of hydrogen was observed by spectroscopists in the late 1800's. The wavelengths of the visible region of hydrogen, called the Balmer series, were found to fit the formula

$$1/\lambda = R \ (1/2^2 – 1/n^2) \qquad n = 3, 4, 5, ... \qquad\qquad (1)$$

where R is the Rydberg constant, with a value of 1.097×10^{-2} $(nm)^{-1}$. Later, other spectral series in UV and infrared were discovered and showed that they all can be expressed as

$$1/\lambda = R \ (1/n_i^2 – 1/n_f^2) \qquad\qquad\qquad\qquad (2)$$

where n_i and n_f are integer numbers. This was an empirical expression and remained unexplained until a quantum model of the hydrogen atom was offered.

The hydrogen spectrum is of particular theoretical interest because hydrogen, having one proton and one electron, is the simplest atom. Neils Bohr developed a quantum theory for the hydrogen atom which explains the spectral lines as resulting from electron transition between energy levels *i* and *f*, Fig. 1. In this model the energy of an electron on the orbit *n* is quantized and express as

$$E_n = - (2 \ \pi^2 \ k^2 \ e^4/h^2) \ (1/n^2) = -13.6/n^2 \ eV \qquad\qquad n = 1,2,3,4 \ … \ (3)$$

where m is the mass and e the charge of an electron, k is the Coulomb constant, and h is the Planck's constant, $h = 6.63 \times 10^{-34}$ J s $= 4.16 \times 10^{-15}$ eV s. n is the quantum number assigned to the orbit. The wavelengths of the spectral lines can be calculated from Eq. 3. If the electron jumps from one orbit, whose quantum number is n_i, to another orbit, whose quantum number is n_f, it emits a photon of frequency

$$f = (E_i - E_f)/h \tag{4}$$

Often it is more convenient to describe this transition in terms of the wavelength of the emitted light. The wavelength can be calculated from Eqs. 3 and 4 and expressed as

$$1/\lambda = f/c = (2 \pi^2 m k^2 e^4 / c h^3)(1/n_f^2 - 1/n_i^2) \tag{5}$$

where c the speed of light in vacuum, $c = 3 \times 10^8$ m/s. A comparison of this result with Equation 1 gives the following expression for the Rydberg's constant

$$R = 2 \pi^2 m k^2 e^4 / c h^3 = 13.6 \text{ eV}/hc \tag{6}$$

When n_f equals 2 and $n_i = 3,4,5 \cdots$, we get the Balmer series,

$$1/\lambda = R (1/2^2 - 1/n_i^2) = (13.6 \text{ eV}/hc)(1/2^2 - 1/n_i^2) \tag{7}$$

or

$$\lambda = (h c) / [13.6 \text{ eV} (1/2^2 - 1/n_i^2)] \tag{8}$$

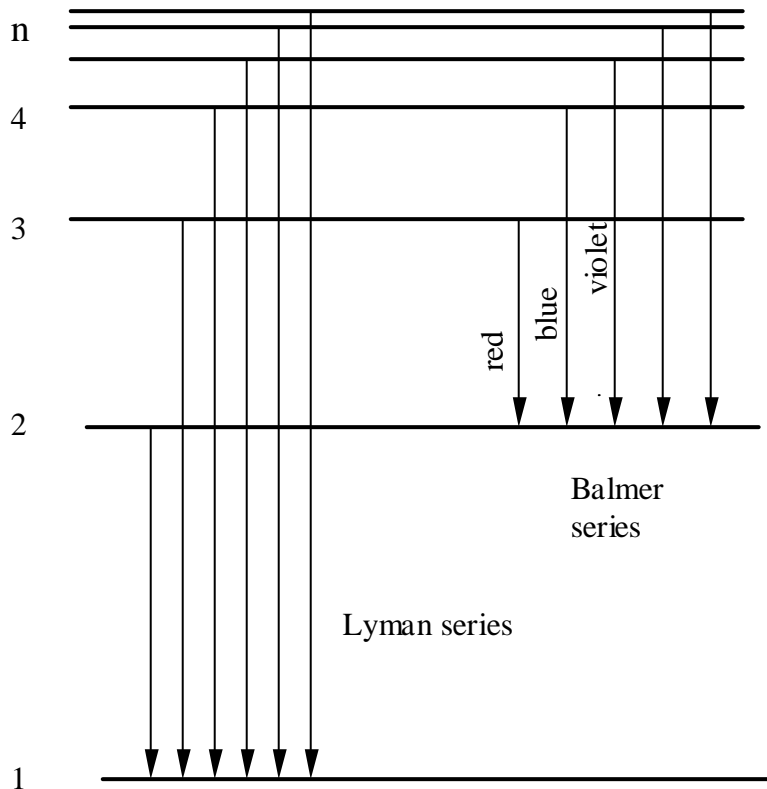For another series of lines, the Lyman series, $n_f = 1$ and $n_i = 2,3,4,5 \cdots$.

Fig. 1. Lyman and Balmer series of spectral lines for atomic hydrogen.

The energy diagram for hydrogen is shown in Fig. 1. Vertical arrows indicate transitions between energy levels. During each transition a photon of frequency $f = (E_i - E_f)/h$ is emitted.

2. **Procedure**

You will measure the spectra using USB4000 spectrometer. The spectrometer measures the amount of light of each wavelength in the sampled spectrum. It consists of a slit, diffraction grating, mirrors, lenses, and a 2048element linear CCD-array detector. All those elements are mounted on a single plug-in computer card. The light emitted by a sample is collected and sent to the spectrometer via a fiber. The USB4000 is a new generation spectrometer of a very small size. But despite its small dimensions it has a remarkable spectral resolution of 1.5 nm and very high sensitivity. It is so sensitive that we will not use a focusing lens to collect the light.

The operating software, SpectraSuite by OceanOptics, controls the data acquisition, provides real-time interface to a variety of spectral processing functions, and displays the results. That software has been installed and configured, and the computer should be ready to start the experiment. Find the SpectraSuite icon on the desktop and double click on it. A new screen with a spectrum, a toolbar and pull down menu will appear. The menu functions, icons, and toolbars are discussed in the section 2.1. Read that section before attempting recording the spectra.

Hold the fiber close to a mercury lamp. BE CAREFUL - OPTICAL FIBER IS MADE OUT OF GLASS AND CAN BE EASILY BROKEN. DO NOT BEND IT ! Observe the spectrum and if it varies widly you may need to hold the fiber more steadily. Adjust the integration time so that the maximum intensity is roughly 3000 counts. Pause or terminate data acquisition to freeze the display. Adjust the y-axis and select the spectral region so it displays only the significant part of the spectrum. If you are satisfied with the spectrum you may print it. You should observe several peaks located at about

436 nm	blue
492 nm	aqua
546 nm	green
577 nm	yellow
579 nm	yellow

With the power cord to the power supply disconnected, replace the mercury tube with the hydrogen tube and record the spectrum of hydrogen. Read the wavelengths of the lines.

Plot a graph of the reciprocal of wavelength -l/λ versus $1/n^2$. Begin the abscissa scale with zero. Draw the best straight line that fits the data points and determine the slope of the line. Note that

$$1/\lambda = R\left[(1/2)^2 - (1/n_i)^2\right] = R/4 - R/n_i^2 \tag{8}$$

has the form of a straight line, $y = mx + b$, with a negative slope equal to the Rydberg constant. Compare the slope of the line with the accepted value of the Rydberg constant. Also, compare the value of Planck's constant obtained from Eq. 6 with the value $h = 6.63 \times 10^{-34}$J s.

2.1.   SpectraSuite

On the desktop locate the SpectraSuite icon and double click on it. The program will start and a new window will appear which will be similar to

Icon that controls the x-axis of the display
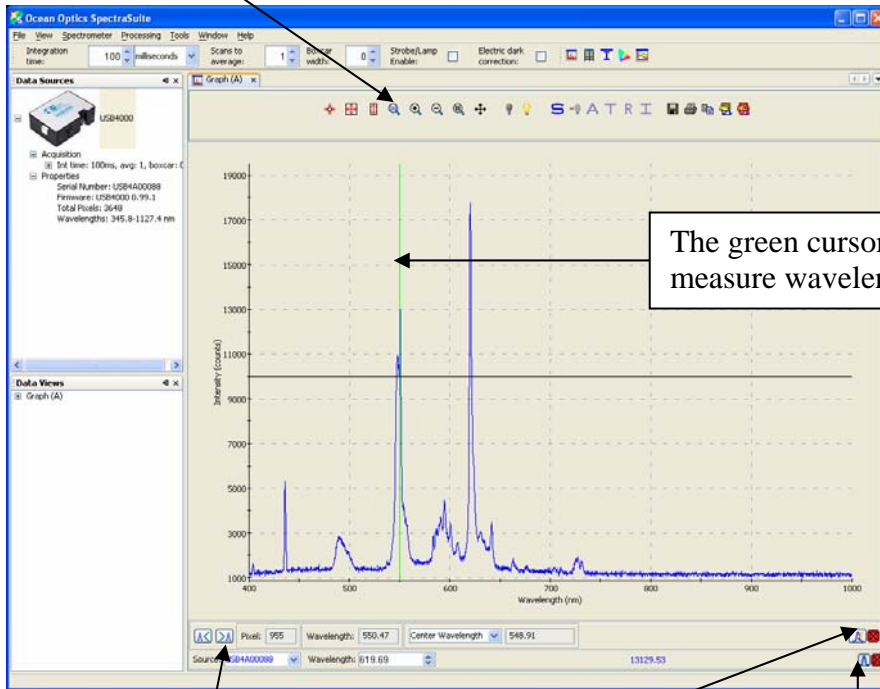
The green cursor used to measure wavelength

To move the cursor

To measure peak positions

Click on this icon to set up peak properties.

The program automatically starts recording spectra. You may adjust the integration time to get a better signal to noise ratio. To stop data acquisition, go to **Spectrometer**, **Acquisition** and then select **Stop**. To resume data acquisition select **Resume**. To adjust the wavelength and intensity range click on the icon that looks like a magnifying glass with numbers 213 inside. Type new range for the wavelengths (400 to 900 is the recommended range) and intensities. When you are satisfied with the quality of the spectrum you may find peak positions. Right click on the spectrum and a green cursor will appear. Simultaneously, below the graph an additional toolbar will appear.

Click on icon which you may find in the right bottom corner of the window and the threshold line (a horizontal solid line) will appear along with a new window that will allow you to control selection of the peaks.
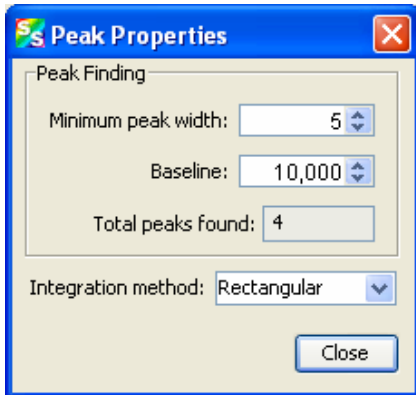
The minimum peak width should be at least 5 pixels and could be as high as 200. Write an appropriate value for the baseline so that only few peaks will actually be measured. You do not want to record all small peaks, most of which are most likely due to noise fluctuations.

Use and to move the cursor to the next peak (left or right). Record the wavelengths of the peaks which will be listed in the box **Center Wavelength**. To print click on the printer icon and you will see a window similar to that:
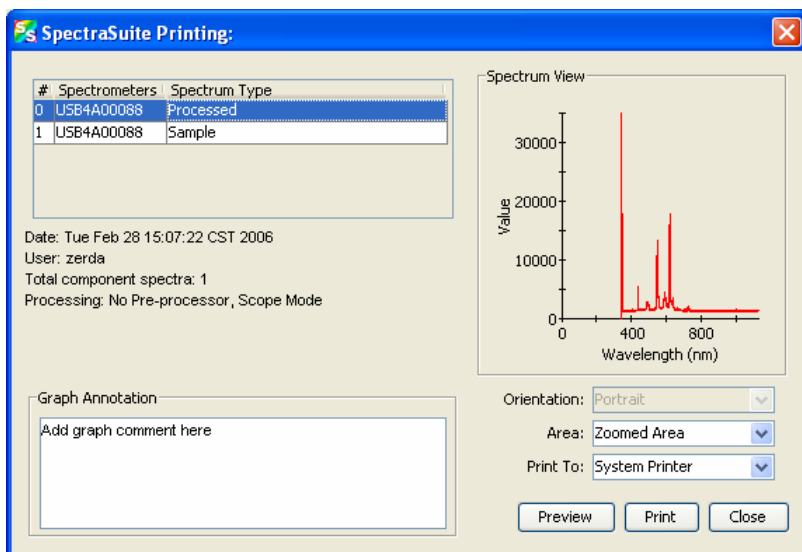
Select the **Processed** spectrum and **Zoomed Area.** You may add comments in the left bottom box; for example, list measured wavelengths of peaks. Select **Print** and follow further instructions. To print you may have to go to the card reader near the system printer to start the process.

To record several spectra on the same page go to **File**, **New**, and select **Spectrum Graph**. A new spectrum will appear. It will have a different color. To stop data acquisition you will have to click on the new spectrum and then go to **Spectrometer** and **Data Acquisition**. Repeat the procedure as many times as needed. To read wavelengths you will repeat the procedure outlined above and to select the proper spectrum go to the box source at the left bottom corner of the graph and pull down the name of the spectrometer. When you select the spectrum the color of the spectrometer name will change.

3.   **Report**

In the introduction discuss the origin of other spectral series for atomic hydrogen (Lyman, Paschen, etc.).

In the report explain how did you make hydrogen atoms to emit light? Hint - in order to see the emitted light, hydrogen atoms were first pumped into excited states of various quantum numbers n.

Atoms do not only emit light but also absorb it. Where do you expect to observe the absorption lines for hydrogen? Suggest an experiment setup to measure absorption spectra.